Math Fundamentals PoW Packet

Teresa's Tiles

March 14, 2011 • http://mathforum.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in January 2003. This is Library Problem #2803. The text of the problem is included below. A print-friendly version is available using the "Print" link from the blue-shaded box on the problem page.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. To access the discussions [log in using your PoW username/password], choose one of these methods:

- use the link to "PoW Member Discussions" from your **My PoW Work as a Teacher** area
- go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526
- from the blue-shaded box, use the **Tips/Ideas from Teachers** link.

Are you making the most of your PoW Membership? If you have an Individual Teacher Membership, consider registering for one of our (free) Orientation Sessions to learn more about the features of your membership. Teachers with Class, School, or District Memberships are welcome to take the free Orientation Session but also are encouraged to register for one of our online courses. View information, dates, and links to register at http://mathforum.org/pd/.

Standards

In ***Teresa's Tiles***, students are asked to find how many tiles Teresa needs to tile her bathroom floor and how many of those tiles will need to be cut. The **key concept** is area.

If your state has adopted the Common Core State Standards, this alignment might be helpful:

Grade 3: Measurement & Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

1. Recognize area as an attribute of plane figures and understand concepts of area measurement.

   b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Grade 4: Measurement & Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

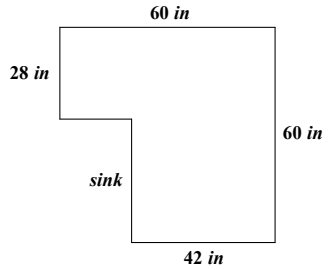
Additional alignment information can be found through the Write Math with the Math Forum service, where teachers can browse by NCTM and individual state standards, as well as popular textbook chapters, to find related problems.

The Problem

Teresa's Tiles

Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.

This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.



Questions: How many tiles will she need to buy to cover her floor? How many tiles will she have to cut in order to cover the entire space?

Extra: What is the size, using whole numbers, of the largest square tile that could be used to tile the entire floor with no cut pieces?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Teresa can tile her floor with 189 tiles. She will need to cut 4 of them in half.

If your answer **doesn't** match ours,

- did you try a simpler version of this problem first?
- did you have 193 tiles as your first answer? Did you realize that both halves of a cut tile can be used?
- did you check your arithmetic?
- is there a part of the description that you don't understand? Did you talk about it with others to get more ideas?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you try the Extra?
- are you confident that you could solve another problem like this successfully?
- did you include each step you took to solve the problem?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- are there any hints that you would give another student?

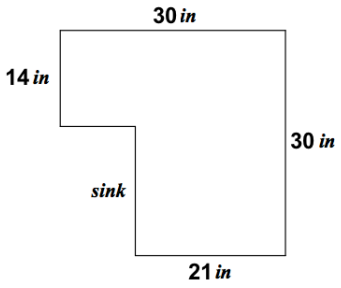
Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Simpler Problem

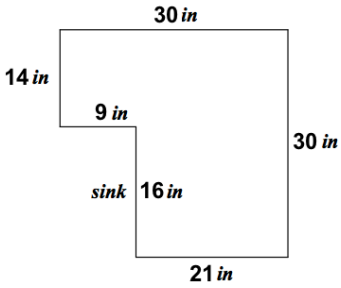
One thing that makes this problem hard are the numbers. I can make a simpler problem to solve by dividing all of the numbers by 2.

I would have a diagram like this:



I would also divide the 4 inch tile by 2 and so I just have to think about a 2 in by 2 in tile.

The top part of the sink area is 9 inches because 9 + 21 = 30. The side part of the sink is 16 inches because 14 + 16 = 30. I add those numbers to my diagram so that I can think about what to do next.



I think about putting the 2 in by 2 in tiles on the floor. 10 tiles will fit across the bottom but there will be one more inch to cover. The 9 in seems like it might be a problem, too, because it's an odd number. If I think of two rectangles then I would have one part that is 30 in by 14 in and another part that is 21 in by 16 in. That would make it easier.

Now I have a plan and I think I should go back to the numbers in the problem.

One rectangle is 28 in by 60 in. If the tiles are 4 in by 4 in I know that 15 tiles will fit across the top and there will be 7 rows of them. That makes 105 tiles.

Another rectangle is 42 in by 32 in. I know that I can cover 40 in by 32 in using 8 rows of 10 tiles each. That's 80 more tiles. That last section is 2 in wide and 32 in high. If I cut 4 tiles I would have 8 tiles that are 2 in by 4 in which would cover the remaining section. In all Teresa will need 105 + 80 + 4 = 189 tiles and four will need to be cut in half.

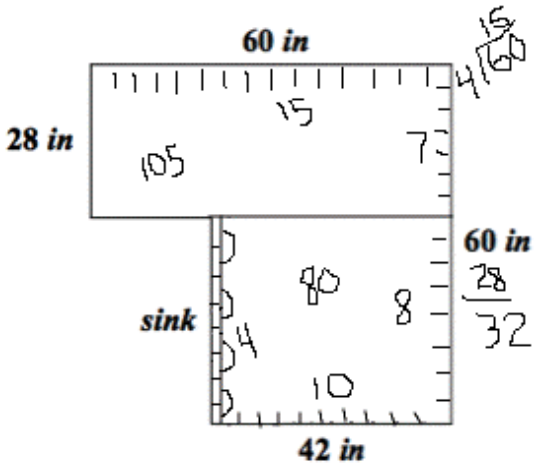
Method 2: Use the Diagram

I used the diagram in the problem to think about how the 4 in by 4 in tiles would fit. First I marked off 4 in across the top. 60 divided by 4 is 15 and so that's how many 4 in tiles would fit.

Next I thought about the side labeled 28 inches and that divided by 4 evenly, too. I marked off 7 and I decided to draw a line, I knew that top part would be covered by 105 tiles.

Since 60 - 28 = 32 I knew that rest of the side could be marked off using 8 tiles. All I had to do was think about the bottom of that rectangle. It was 42 in and so I marked off 10 but knew that there was a 2 in wide strip. That was where I would have to cut! I thought about the 8 pieces I would need and knew that would be 4 tiles each cut in half.

When I added up the number of tiles, I got 189.



Method 3: Calculate Area

In order to find the area of Teresa's bathroom, I first divided the bathroom shape into two rectangles by thinking about drawing a horizontal line across to make two rectangles of the shape. To find the area of these two rectangles, I multiplied length by width of each.

top rectangle: 60 in • 28 in = 1680 in^2

bottom rectangle: 42 in • (60 in - 28 in) = 1344 in^2

The 4 in by 4 in tile has an area of 16 in^2 . If I add the areas of the two rectangles and divide by 16, I should have an idea of how many tiles Teresa will need.

1680 + 1344 = 3024

3024 ÷ 16 = 189 tiles

The problem, though, also asks about how many tiles Teresa might cut. I looked at the bathroom shape again to see how the tiles would fit.

I looked at each of the sides and divided by 4 to see how many perfect tiles would fit in the columns or rows.

top rectangle: 60 ÷ 4 = 15 and 28 ÷ 4 = 7
She would need 7 rows of 15 tiles or 105 whole tiles.

bottom rectangle: 42 ÷ 4 = 10.5 and 32 ÷ 4 = 8
She would need 80 whole tiles (10 • 8) and 8 halves or 4 whole tiles cut into halves (1/2 • 8).

If I add those numbers of tiles I get 105 + 80 + 4 and again I have 189 tiles but now I also know that Teresa will need to cut 4 of the 189 tiles so that the tiles will fit on the floor.

Extra: To find the whole number of the largest square tile that could be used to tile the entire floor with no cut pieces, I used greatest common factor (GCF) method on the numbers 60, 42, 32, 28, and 18. The largest tile would be 2 in by 2 in.

Method 4: Calculate the Entire Area

As I looked at the diagram in the problem I thought about the entire area of the bathroom, including the sink. The shape is a 60 inches by 60 inches square. The area of that square is 3600 in^2. A tile that is 4 inches by 4 inches has an area of 16 in^2. If I divide 3600 by 16, I get 225 tiles.

This is not the final answer, though, because I still have to find the area of the sink to subtract from the 225 tiles.

The length of the sink is 32 inches and the width of the sink is 18 inches. That's 576 in^2 of area that needs to be subtracted from the 3600 in^2 or if I think of it in terms of tiles and first divide by 16, I need to subtract the 36 "sink" tiles from 225. I now have 189 tiles.

As I looked at the sink section I thought about how many times 4 goes into 18 and 32, because Teresa was using 4-inch tiles. I wanted to think about how they might fit. I concluded that 4 goes into 32, 8 times evenly, and 4 goes into 18, 4 times with a remainder of 2, or 4 1/2 times. Teresa would have to cut 4 of the tiles in front of the sink because the 4-inch tiles can't fit exactly into that 18-inch side, which means that she would need to use eight 2-inch tiles to cover that area.

Extra: The largest tiles Teresa could use without having to cut any is 2-inch tiles. I found this answer because 2 is the largest common denominator of the lengths and widths of the bathroom.

Method 5: Change the Representation

The main mathematical idea is area.
The relationships in this problem is to think about how the 4 by 4 tile will fit over the bathroom floor.

We could represent this problem by

- drawing a picture - We can use the drawing in the problem but add to it
- drawing a series of pictures - We can start with the drawing in the problem and as we think about things we could draw another picture and another
- acting the problem out - We could use tiles in the bucket we have in our classroom to think how this might work
- organizing quantities in a table - If we divide the shape into parts we could keep track of the different areas in a table.

We talked about the best idea and we decided to get the bucket of tiles and act the problem out. We pretended that the small tiles were really 4 inches by 4 inches. We counted out 10 knowing that would be 40 and then we needed 5 more to make 60 inches going across the top. Next we thought about the part of the picture labeled 28 in and we knew that would be 7 tiles. We didn't fill them all in but we knew that if we did it would be 7 rows of 15 or 105 tiles.

The right side was labeled 60 in and we already had 28 in covered using the 7 tiles so we knew we needed 8 more. Now we thought about the bottom part labeled 42 in. We counted out 10 tiles knowing that would cover 40 inches. What about that last 2 inches? We knew that if we cut the 4 in by 4 in tiles so that we had 2 in by 4 in pieces, that would fit. If we cut 4 tiles we would have the 8 remaining pieces that we needed to cover the floor. We added 105 + 4 + 80 and it equaled 189 tiles.

Teaching Suggestions

When we first offered this problem most submitters broke the irregular space into two rectangles, figured out how many tiles could fit along each side, and then calculated the area in tiles for the two rectangles.

One of the twists in this problem was that you couldn't simply find the area in square inches, but you had to figure out the area if your unit measure was a tile that was 4 inches by 4 inches. An additional twist was to figure out which tiles needed to be cut in order to fill the narrow space in front of the sink. Some students added in lines so that they could calculate the area of the bathroom and then subtracted the area of the sink. Those who then divided by the area of one tile (16 square inches) seemed to have a good strategy except they didn't have a way to think about the cut tiles. Of course, those students who realized that they had not considered the cut tiles, could go back and figure out where the cut tiles would need to be placed.

One idea that we talked about in the office recently is that it's possible that a student might think that only one of the halves of a cut tile could be used – or that the likelihood of cutting a tile well enough to use both halves was slim. For example, one half would be useable but the other would be cracked or, in other words, the cut would not be clean enough to produce two perfect halves. We decided that this would be a great reflection and if a student were to explain this line of thinking, we would accept an answer of 193!

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Using the Noticing/Wondering activity might help students notice what is happening in this problem. Encourage them to actually make a list of those noticings. Writing them down is an important part of the process. Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are several ways to approach this problem. And keep in mind that we may not have thought of them all!

I also encourage you to explore activities in the Change the Representation and the Solve a Simpler Problem strategies in our Activity Series. Something there may provide the impetus that your students need.b

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the Current Problems as we add them throughout the 2010-11 season, including a calendar, consider bookmarking this page (a link to the page is always available in the left menu when you're logged in):

http://mathforum.org/pow/support/

Sample Student Solutions

focus on **Interpretation**

In the solutions below, I've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column. For example, might answer only one of the two questions.	Understands that • they should be looking for the total number of tiles and how many will need to be cut. • the figure represents an area to be covered. • the entire area must be tiled. • the sink area will not be tiled.	Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra.

BrownEyes10
age 10

Interpretation
Novice

Answer is 16,800 tiles.

We got this answer by multiplying the lengths of the open space and then multiplying that answer by 4 sides.We got the answer.

When I see a number that large in an answer, I always have the inclination to ask about costs! I might ask this team of students to imagine if a tile cost $1 or $2 or even more – how much would it cost to tile the bathroom? Is $16,800 or $33,600 reasonable? I hope not!

Next, though, I would need to ask them a question to get them back to thinking about the problem. I might ask them what they notice when they look at the diagram. I might ask them why they multiplied since if they respond that you multiply to find area, which would be a good starting point for more conversation.

Marissa
age 10

Interpretation
Novice

My anser is 190

I got my anser by adding 28+60+60+42=190

I notice that the Marissa added the four numbers visible on the diagram. I would point out to them that it looks like they might have been trying to find the perimeter of the shape, the measurement around the outside of the bathroom.

I might ask them to think about how many rows of tiles Teresa would need to fill *the space. Instead of focusing on just the labeled numbers, they may start using the diagram to think about how the tiles would fit.*

Matt, Gill, Diana
age 13

Interpretation
Apprentice

Teresa will need 189 square tiles. She won't need any fractional tiles.

First we found the total area of the bathroom by breaking it into two rectangles. The total area was 3,024 square inches. Then we found the area of each tile, which was 16 square inches each. So then we divided the area of the room by the area of each tile (3024/16)and we got the answer to be an even 189 tiles with no fractional pieces.

This group's solution is typical of students who think only in terms of area calculations and not about how the tiles will fit within that area. The numbers don't tell the entire story in this case.

I might ask them how they knew there were no fractional pieces. Were they expecting the division problem to result in a fractional answer? Did they consider how those 189 tiles might fit into the space?

Patrick
age 12

Interpretation
Apprentice

My answer for the questions was she needs 756 4in by 4in tiles to fill her bath room. She will have to cut 7.5 tiles to fill it right.

I figured out the first question by spliting the room into two rectangles, a 28" by 60" and a 42" by 32". then I found the area of each rectangle.

60"x28"= 1,680/4=420 tiles

42"x32"= 1,344/4=336 tiles

Then I added them together to get 756 tiles.

To get the second answer I looked to see how many tiles went across each of my rectangles and added them together to get 7.5 tiles.

Patrick's solution illustrates another common misconception for this problem. Students divided by 4 instead of 16 forgetting that they needed to divide the area of the room by the area of the tile to find the number of tiles.

It's great that he remembered to think about how the tiles will fit. I would ask him to explain that part so that I could understand how he decided Teresa would need to cut 7.5 tiles.

Jay
age 10

Interpretation
Apprentice

She will need 11.875 tiles which means she will need to cut 1 tile

I added 60in+60in+42in+28in=190in then I multiplied 4x4=16 then I divided 190 by 16 and I got 11.875.

I notice that Jay is thinking in terms of perimeter when he talks about the bathroom floor but he seems to be thinking of area when he talks about the tile.

I would let him know that the area of the 4 in by 4 in tile is 14 square inches because 4 rows of 4 would be 16. It might help him think more about area of the idea of what it really is can be reinforced.

Next I would wonder why he added those other numbers since that's thinking more about perimeter than area.

Pat, Dave, Toni
age 13

Interpretation
Apprentice

268 complete tiles plus 15 cut

first we split the shape into 3 sections 28 by 18 , 60 by 28 ,and 42 by 60. then we found out how many times 4 went into all of those numbers and then we found out how many tiles where extra and that was 15 that we needed to cut and then we added up all 3 total amounts of tiles together and we got 268.

I notice that Pat, Dave, and Toni have three rectangles. At first I thought they had made a vertical line to get the 28 x 18 and the 42 x 60 and then also counted the sink area but that third rectangle would have been 18 x 32. As I looked at the diagram again, I see that they counted the "top" section twice – once drawing a vertical line and another time drawing a horizontal line.

Besides asking them about that, I might ask them why they divided by 4 just to reinforce the idea of the area of the tile that should be considered.

Nadine
age 11

Interpretation
Practitioner

You need a total of 189 tiles to cover the whole bathroom floor. You also need to cut 4 tiles in half. *If you were smart...you would buy about 190 or 200 incase you make mistakes cutting!!

First, I took away 28 from 60 in. to figure out how many inches the bottom left side was. I got 32 in.

Then, I drew a line to seperate the 32 from the 28, making into 2 boxes: a top and a bottom one. I knew that the other side of the bottom box would be 32 as well.If the bottom of the little box is 42 in, then so is the top.

I multiplied the base and hight of the little box to get it's area. It was 1344. Then, I knew that if each side of the tiles was 4 in, then it's area would be 16. So then i divided 1344 by 16 and got 84, or 84 tiles.

Nadine understands the problem completely but my favorite part of her solution is her reflective comment. How true it is to recommend buying some extra just in case there are errors in cutting!

There are some issues with Nadine's use of units. If she revised and paid attention to linear units vs. square units her Clarity score could improve.

Then, I went up to the bigger box. The bottom line is 60 in, and the hight is 28 in. I multiplied them to get the boxes area and got 1680 in. I divided it by 16 and got 105, or 105 tiles.

Then, I figured out that if the hight was 28 in, then there would be 7 rows because when tou divide 28 by 4, you get 7. (4 because that is how big the tiles are) Then i divied 60 by 4 and got 15, and I knew that there would be 15 tiles in each row.

Going down to the little box, I knew I had to divide 32 by 4. I got 8, so i knew that there would be 8 rows. Then i realized that i cant divide 42 by 4. I did know that 4 went in to 40 ten times. You can fit ten tiles in the rows, but then there are 2 inces left. So then you have to cut a tile in half because it is 4 in. and we need 2.

A total of four tiles must be cut because there are 8 rows and we need to cut the tiles in half. I also added up the tiles for the two boxes, (105 and 84) to get the total amount of tiles, which was 189.

Alexis
age 10

Interpretation
Practitioner

To cover her entire floor, Teresa will have to buy 189 tiles and she will have to cut 4 tiles in half.

Before I got started, I read the problem carefully. By doing this, I prepared myself to know what to do in the problem. I decided that in this problem I would have to do mainly multiplication and division. Then I searched for some important information. I thought these things were most important:

1)The tiles are 4 inches on each side.
2)The sink area does not get tiled.

After I concidered the information, I got to work. First, I had to see how big the room was (so I had to use some prior knowledge). To do this, I multiplied 42 and 60 (because if you cut off that extra space with the 28 inches those are the two demensions) and it came out to 2520 square inches. The equation looked something like this:

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  42
 *60
----------
 2520
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Then, I multiplied 28 (other extra part) and 18 (inches I subtracted earlier to make work easier) and that came out to 504 square inches. That equation looked like this:

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  28
 *18
----------
 504
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The next step was really easy! All I had to do was add the two answers from the multiplication problems together. My equation looked something like this:

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 2520
+ 504
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 3024
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So my answer so far is 3024 square inches with only 2 more steps to go!

The next step was to divide 3024 by 16 because 4 (in. on one side)*4 (in. on the other) =16. My equaition looked something like this:

3024/16= 189

So now I have an answer of 189 tiles, but the final step will change that.

The final step was a little complicated. On the side that's 42 in. you will realize that 4 does not go into 42 evenly, but it can go in 10 times with a remainder of 2 inches. If this is true then you will put one 2 inch tile along that wall, but, as you will see if you draw

Alexis, like Nadine completely understood the problem. I've included this solution to illustrate the result of a student using feedback from a mentor (their teacher, for example!) and revising until they reach expert level scores in Completeness, Clarity and Reflection.

Alexis revised four times and with each draft she improved her solution.

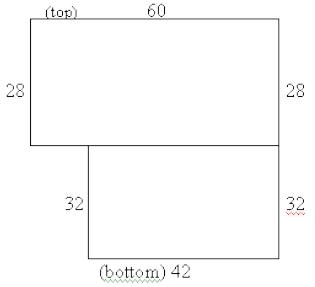
it out, you have to fill in the inside of the floor up, not just the outside! I drew it out and I realized that to cover up the entire space Teresa will have to cut 4 tiles in half making eight 2 inch tiles!

Becca
age 10

Interpretation
Expert

She will need to buy 189 tiles to cover her floor. She will have to cut 4 tiles. Extra: The largest square tile that could be used to tile the entire floor with no cut pieces would be a 2 inch tile.

First I drew a line across the floor to make 2 rectangles. (See picture)



The top rectangle was 60" by 28". Since 4 goes into 60, 15 times, I knew I could fit 15 tiles across the top rectangle. I also knew that I could fit 7 tiles going down because 4 goes into 28, 7 times. To find how many tiles I needed for the top rectangle, I multiplied 15 by 7 to get 105, which is the area.

I knew that the distance from the top of the floor to the bottom was 60" and that the distance from the top to the sink was 28". I figured out that the distance from the sink to the bottom was 32" because the total distance had to be 60" (60 - 28 = 32). 32 divided by 4 = 8 so I could fit 8 tiles going down the bottom rectangle. Going across the bottom rectangle I could fit 10 ½ tiles (42 / 4). So, the total number of tiles needed to cover the bottom rectangle would be 8 x 10½ = 84.

The total number of tiles needed to cover the floor would be 105 + 84 = 189. Since 8 rows in the bottom rectangle have 10 ½ tiles, then she would have 8 half tiles and that would mean she had to cut 4 whole ones.

Extra: I made a factor tree of all the dimensions.

60 = 2 x 2 x 3 x 5
28 = 2 x 2 x 7
32 = 2 x 2 x 2 x 2 x 2
42 = 2 x 3 x 7

Then I found the biggest common denominator, which was 2. So, the largest square tile she could use with no cut pieces would be a 2" tile because 2 goes into all the dimensions evenly.

Becca has not only done a thorough job on the main question but has included an explanation of how she thought about the Extra question.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Suzanne <suzanne@mathforum.org>