Problem Solving and Communication Activity Series

Change the Representation

All math problems, whether they are word problems, arithmetic problems, equations to solve, etc., come to us in a particular representation. Word problems are represented in story form, using words and often referencing a particular context. Arithmetic problems are represented numerically. Equations are represented using mathematical symbols. Each representation has benefits to the problem solver. For example, word problems allow students to apply their knowledge of the given context, which can allow them to check that their approaches are reasonable. Numeric and symbolic representations can make it easy for students to manipulate objects in the problem, and to quickly see patterns. Visual and physical representations, such as manipulatives, diagrams, and graphs, can often help students gain new insights into the problem and provide them with additional tools for solving it. Changing the representation can mean use of a different form of representation (e.g. using a line drawing for a word problem) or it can mean trying different ways of presenting the information in the same form (e.g. rewriting all of the numbers as fractions with a numerator of 1). Considering multiple representations and choosing representations that fit the problem well are important problem-solving skills.

The activities below help students to brainstorm and work with multiple representations, and compare what they learned about the problem using their different representations.

The activities are written so that you can use them with problems of your choosing.

Problem-Solving Goals

Changing the representation of a problem can help problem-solvers:

- Strengthen their understanding of the problem.
- Gain new insights into the problem or solution.
- Provide additional tools for solving the problem.
- Find multiple solution paths, leading to deeper mathematical connections.

Communication Goals

Changing the representation of a problem requires that students change how they communicate about the problem and find different ways to express the same idea or information. They might:

- Paraphrase the problem in terms of a different representation.
- Re-tell the story of the problem with a different context.
- Organize the numerical and calculation strategies using a table or other organizational method.
- Use mathematical symbols to restate the problem succinctly.
- Use diagrams to communicate the math in the problem.
- Represent the problem graphically.

Activities

I. Brainstorming Representations

Format: students working individually or in pairs, then sharing with groups of 4-6.

There are many ways to represent math problems and mathematical ideas. Math problems are often represented in words. Math can be represented visually, through graphs, diagrams, and sketches. Tables and expressions can be used to represent math ideas numerically. Mathematical ideas are often represented symbolically, with operations, numbers, variables, and functions. Each representation can help you understand and solve the problem in different ways. Problems represented in words help you to make sense of the problem and use your knowledge of real-world situations. Graphical representations can lead to new insights or problem-solving methods. Numerical

representations can help you find patterns and generate strategies. Symbolic representations represent math ideas clearly and succinctly, and help you to manipulate the mathematical objects.

Sample Activity

Work individually or in pairs to begin filling in the blanks of the following prompts and choosing representations for just a few minutes. Then share ideas with the larger group of 4-6 students. The first question asks you to think about the math ideas in the problem, which might get you thinking about other representations you know. The second asks you to think as creatively as you can.

1) The main mathematical ideas and relationships in this problem are _____.

2) I could represent this problem by (choose as many as you think would be helpful):

- o drawing a picture,
- o drawing a series of pictures (like a comic strip),
- o acting the problem out,
- o building a 3-d model,
- o representing the problem using blocks, counters, etc.
- o representing the problem using a number line or lines,
- o making a graph or graphs,
- o organizing quantities in a table,
- o writing relationships as equations,
- o telling a different story,
- o writing the numbers or expressions in a different way, like:
  - o rewriting the fractions to have common denominators, or as improper fractions, or as mixed numbers, etc.
  - o simplifying equations or expressions,
  - o expressing all of the quantities in terms of the smallest item,
  - o using expressions that have the same values but make calculations easier, like changing .375 to 3/8 or changing x/2 to 0.5x,
  - o changing how radicals or exponents are represented (e.g. changing square root to raising to the 1/2 power),

- o some other way: _____.

Share your thoughts with your group. If the ideas you hear spark other ideas, record those too. Try to brainstorm as many possible representations as you can.

Key Outcomes

- Identify key mathematical ideas in the problem that can be represented.
- Think of ways that mathematical ideas are sometimes represented.
- Generate multiple possible representations for the problem to be solved.

II. Representing

Format: Students working in pairs.

The focus in this activity is on using writing to organize the problem solving activity, to notice patterns or ideas that make the solution possible, and to ask specific questions that need to be answered in order to make progress,

Sample Activity

Step 1: Look over the list of possible representations that your group brainstormed. Pick one that you have an idea about how to get started on and try it. If you're not sure about any of them, ask a groupmate to show you their first step and see if that helps you get started.

Step 2: Work on any ideas that come to you as you try a representation.

Step 3:

If you get stuck, share your work with a partner and ask them to help you by:

o seeing if your representation gives them any new ideas

o seeing if they can spot any quantities, shapes, or relationships that you haven't included or you accidentally changed

o helping you brainstorm another representation to try

If you find an answer, check your work by trying another representation and/or comparing with a partner.

Key Outcomes

• Play out a particular representation as fully as possible.
• Pay attention to quantities and relationships across representations.
• Work with others to see multiple perspectives and fresh ideas.

III. Comparing

Format: Students working individually or pairs and then sharing with the whole group.

Sample Activity:

Step 1: Within your group of 4-6, compare the different solutions you got. Do they agree? If any solutions disagree, look closely at the representations, and check:

   o Are all of the quantities or shapes in the problem represented?

   o Did any of the quantities or shapes get represented in a way that changes them (e.g. the problem is about a general rectangle but you drew it as though it was a square)?

   o Are all of the relationships in the problem represented?

   o Did any of the relationships get represented in a way that changes them?

Step 2: Once you are confident of a solution, compare the different representations you used. Did different representations lead to different insights? In what ways are the different solutions similar?

Step 3: Select the representations that provided the most insight into the problem or key steps in the solution. Prepare a presentation for the class on how each representation helped you get to a key insight into the problem.

Step 4 (optional): Submit your write-up to the PoW online.

Step 5 (optional): Use a jigsaw or gallery walk format to share your explanation with classmates and to appreciate their insights.

Key Outcomes:

• Compare insights generated by multiple representations. Identify the similarities and the differences in the contributions from each representation.
• Evaluate how well different representations fit a given problem. Figure out how to recognize when a particular way of changing the presentation of the problem will be useful.