The Math Forum: Problems of the Week

Problem Solving and Communication Activity Series

Guess and Check I

Guess and check is an important (and popular) problem-solving strategy, though it often gets a bad rap and may not be developed into the strong and powerful resource it could be. The guess and check strategy has at least three purposes: (1) to understand a problem thoroughly, (2) to home in on a solution, and (3) to discover efficient ways to jump to a solution by noticing patterns and developing related algebraic representations.

In order to get better at Guess and Check, students need to get better at understanding the quantities in the problem and the relationships among them, representing and organizing those relationships, and looking for patterns. Those skills are at the heart of almost any problem-solving strategy, and are necessary prerequisites for writing equations and other mathematical model work. We strongly encourage you to spend time helping your students get good at guessing, calculating, and checking to help them become better overall problem solvers.

The activities in this round help students figure out what it means to get better at guessing, calculating, and checking, as they compare each others' guess and check strategies and learn to ask critical questions of each other.

Problem-Solving Goals

Guess and check strategies can help problem-solvers:

- More fully understand the problem.
- Find patterns.
- Discover and generalize important relationships.
- Solve complex problems.

Communication Goals

Guess and check strategies are enhanced when students focus on writing to learn. Good problem-solvers record their works in ways that:

- Make clear what they guessed, what calculations they did, how they checked their guesses, and what did or did not work.
- Make it easy to notice patterns and repetition.
- Help them analyze their attempts so that they can make their next guess even better.
- Allow them to share their guesses, their calculations, and their checks with others.

Activities

I. Guess, Calculate, Check

Format: students working in small groups (3 students to a group).

We encourage all students to begin problem solving by using some form of the *Understanding the Problem* strategy covered in the first two rounds of the Activity Series, but *Guess and Check* can be another good method for familiarizing oneself with a problem.

One of the first requirements of the guess and check strategy is to determine the quantities or aspects of the problem they can "play with." This might be equivalent to "Doing it Wrong" or noticing quantities and relationships between quantities in *Understanding the Problem.* Once they have chosen an aspect to guess about, students need to see the effect of the guess on as many other quantities and relationships as they can. On the way, they will also think about how to check their guesses.

Sample Activity

Each group will generate 3 guesses. For each guess, students will identify and perform the necessary calculations to check their guess. Each student will have a chance to be the group's scribe for one guess. The scribe should make sure to include as much as they can of:

- what was guessed and why,
- the calculations that were completed based on that guess (it should be clear what the results of each calculation represent), and
- how the guess was checked.

One reasonable outcome from this effort may be for the students to get clear about what they may not understand or need to first figure out before they can effectively guess and check. Some students may have trouble doing calculations, thinking of ways to check guesses, or even understanding what to guess. In that case, they should spend the time listing what they do know and what they need to figure out. They might return to one of the *Understanding the Problem* strategies, or find other resources in order to figure out these sub-problems.

Key Outcomes

- Focus conversations and writing on three distinct aspects of the solution path (guesses, calculations, checks).
- Develop awareness of where difficulties arise as students guess and check; is it deciding what to guess. figuring out the calculations, or checking guesses?
- Generate record keeping methods for guess and check that can evolve into organized and systematic approaches.

II. Class Discussion on Guess, Calculate, and Check

Format: small groups present their guesses; the audience has the task of asking certain kinds of questions and/or giving certain kinds of feedback (Herrenkohl & Guerra, 1998).

Having the opportunity to present their own work, see others' work, ask questions of others, and discuss similarities and differences will help students become aware of their own guess and check process. With each presentation they should note:

- Clarity of organization and labeling.
- Completeness of the work.
- Similarities and differences among groups.

In order to facilitate discussion and keep all students engaged, in the sample activity, we have provided roles for audience members to take as they listen to groups.

Sample Activity

Step 1: Assign each group member one the following roles for while they are in the audience. If desired, give them the appropriate handout (see appendix) for their role. Students should sit with the other people assigned to their role.

- Noticing guesses: Was it clear what guesses were made and how they relate to the desired answer? Can you tell how the guesses were generated? Once you've seen multiple groups' presentations, do you notice similarities or differences among the guessing strategies?
- Noticing calculations: Are the calculations organized or labeled so you can tell what is being calculated and how? Are all the necessary calculations shown? Are there calculations that are not necessary? Do you notice patterns among the calculations? Once you've seen multiple groups' presentations, do you notice similarities or differences in the calculations that are being done?
- Noticing checks: Can you tell what constraints in the problem were used to check guesses? Are all the necessary constraints used? Do unnecessary or added constraints exist? Did you notice similarities and differences among the constraints that different groups used?

Step 2: The first group presents their guesses, calculations, and checks. As they present, each audience member jots notes to him or herself based on the questions assigned to their role (noticing guesses, calculations, or checks).

Step 3: After each presentation, the people within each role get together and decide what clarification questions they would like to ask the group, what noticings they would like to share, and what they are wondering about. At this point, group members should listen (and write down things they would like to remember, but not respond)

Suggestions for helping students turn less helpful comments into useful questions, noticings, and wonderings

Cheerleading

- "I thought you did a nice job" – Could you tell me one specific thing you noticed and liked about their work?
- "It was good that you made a table" – What did you notice that is useful about the table? How does it help them improve their guesses?
- "Your work was neat" – Could you be a little more specific? What's one thing about how they organized their work that you noticed is helpful?

Vague Suggestions (In this section and the next our facilitation comments often combine both noticings and wonderings, but often it works best to just start with the noticing and give the recipient a chance to think about it and possibly figure out some implications for themselves first.)

- "I think you should make a table" – Why do you think they should do that? Can you restate that as a more specific wondering based on what you noticed? 'I noticed that your guesses were all spread out, and it seemed like you had to… and I wondered if a table would make it possible to . . .?'
- "You need to be clearer" – Is there a specific thing they could change? Can you restate that as a wondering based on what you noticed and what you think would happen if they were clearer? 'I noticed that it was hard to . . . and I wonder if it would help to . . . so that you can see . . .'

Unhelpful Criticism

- "Your work doesn't make any sense" – What's one thing you (or anyone) noticed that this group did well? Could you give them a suggestion that builds on that? Is there something you would like them to explain more? 'I did not understand what you did when . . . and I wonder if it would help to . . . ?'
- "Why didn't you just get to the answer, if you were that close?" – This group's work is probably well organized if you can see how to jump to the answer and that would be worth noticing and appreciating. Could you explain what pattern or fact you're seeing in their work? "I notice that the way you have it organized, I can see there's a pattern like . . . and I wondered if you noticed that and thought about how to use it?'
- "You did the check wrong" – Could you be more specific in what you are noticing? 'I noticed that in your check you … and I wondered whether that makes sense because . . .'

Key Outcomes

- Develop a better understanding of the problem, the mathematical relationships that must be recorded and carried out, and what counts as an answer.
- Gain perspective on the guess and check process; what parts were difficult for you? Were your calculations and checks complete?
- Improve your recording of your thinking; notice different ways of recording guesses, calculations, and checks.
- Identify multiple ways to guess, and how they effect calculations and results.

III. Improve

Format: students form new groups of three, working with members from other groups. Each group will report back to the larger group.

Students have two goals in this activity. The first is to try to make better guesses by observing the results of previous guesses, which might include looking for ways to jump straight to a solution, using patterns or algebraic representations. The second is to be aware of how different ways of keeping track of guesses, calculations, and checks make it easier to notice and analyze patterns that lead to improved guesses.

Sample Activity

In your small group, think about the feedback that you got when you presented your ideas. Decide how you will improve your guesses and your ways of organizing or recording your work.

- Are you getting closer?
- What did you notice about each guess that helped you decide how to improve it? (i.e., Did you notice if you were too high or too low? Did you notice how far off you were?)
- Did you notice patterns in what changed as a result of each guess? Did your new guess get you get closer or further by the same amount as the last guess?

- Are there ways to reorganize you work to see patterns more clearly?
- Did you notice doing certain calculations over and over?

Prepare to report back to the class on how the group's method of recording helped you figure out how to improve the guesses, calculations, and checks, even enabling you to find a calculation or equation that you can use to solve the problem directly; was there something you would improve about how you recorded your calculations for next time?

Some techniques you might suggest if students are having a hard time improving their guesses:

- Did you try putting your guesses in order?
- Did you try making smaller and larger guesses? What effects do you notice?
- Did you see what effect going up or down by the same amount has?
- What parts of your calculations are changing? What parts are staying constant?
- Could you highlight or somehow show which number in each of your calculations is the guess?
- Would a spreadsheet help you reorganize your guesses?

Key Outcomes

- Recognize how to change guesses in the right direction; in some problems, this can lead to a more efficient solution.
- Gain a sense of control over the guesses, getting closer and closer to a solution that is more strategic.
- Develop effective methods for recording guesses, calculations, and checks that comes from students' own noticings of what is useful.

References

Herrenkohl, L. R. & Guerra, M. R. (1998). Participant structures, scientific discourse, and student engagement in fourth grade. *Cognition and Instruction, 16(4),* 431-473.