



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Guess and Check II

In our previous round of *Guess and Check*, we focused on using guess and check to understand the problem and to home in on a solution. In this round we delve deeper into the uses of guess and check. We present three different activities that focus on students at different levels of development as problem solvers or different levels of understanding of this round's PoW. You might use different activities with different groups of students to differentiate problem-solving instruction.

Activity I, Getting Unstuck helps students who aren't sure where they're stuck, but don't have a way to approach the problem. Using the familiar language of guess and check to figure out where they are stuck helps students find entry points into a problem. Figuring out what to guess, carrying a guess through a problem, and testing the result is a powerful way to focus on relationships and deepen understanding. For more *Getting Unstuck* activities, see Round 16 <http://mathforum.org/pow/support/activityseries/getunstuck.html>

Activity II, Testing 1, 2, 3, 4 helps students who have a possible solution to a problem but aren't sure how to test that solution. When students say, "I'm done, did I do it right?" this activity provides concrete ways for them to answer that question for themselves.

Activity III, Hypotheses helps students who are getting good at understanding problems but need support as they move into the planning and strategizing phase. This activity can be used with highly proficient students who are ready to step back and think more generally about the type of problem. Specifically, it helps students slow down and articulate what they want to try and why; this habit helps students be more reflective and get unstuck if their plan doesn't work out. For more *Planning and Reflecting* activities, see Round 15 <http://mathforum.org/pow/support/activityseriesplanandreflect.html>

The activities are written so that you can use them with problems of your choosing. The activities in this round are not sequential; it is likely that you will want to assign only one of the activities to a particular group of students.

Problem-Solving Goals

Guess and check can help problem solvers:

- Identify where they are stuck.
- Ask good questions.
- Test the reasonableness and accuracy of a solution.
- Make inferences about good strategies to use and try those strategies out.
- Identify key information and relationships, and guess at how to make use of that information.

Communication Goals

Students using guess and check can use writing to:

- Organize information to see patterns.
- Connect their words to mathematical expressions.
- Represent work succinctly and mathematically.
- Communicate where they are stuck in problem solving.

Activities

I. Getting Unstuck

Format: Students working alone, then pairing up.

Often we get stuck when we are solving problems. The key to getting unstuck can be asking a good question. Guess and Check can be a way to help you figure out where you're stuck and ask a good question. Sometimes just coming up with the right question will make it possible for you to answer yourself; other times, it will make it possible for someone to help without just showing you what to do.

Sample Activity:

Step 1 (diagnosis): Figure out where you are stuck. Which of these fits your situation?

Guess stuck

- I don't know what to guess. I don't even understand what the problem is asking.
- I don't know what is a reasonable guess to make.

Calculation stuck:

- I made a guess but I don't know how to use it to see if it works. What calculations can I do, based on my guess?
- I could check my guess, but I don't know how to do this calculation.

Check stuck:

- I tested my guess and did some calculations, but I don't know what that tells me. How can I tell if I'm right?
- How can I tell if I got closer? How do I figure out if my results make sense?

Step 2 (questions): Explain what is hard or confusing to you and ask a specific question about that. The goal is to ask a question that gets you just enough information to make more progress but doesn't give you more than you really need. Your explanation and question might sound like one of the following:

- I know that I have to figure out ___ and ___ but which one should I use for my guess?
- I guessed ___ but then when I tried it I had to figure out _____. How do I do that?
- I guessed ___ and I got _____. Looks like it works, but I don't know how to be sure that it's right. Is there another piece of information in the problem that I should be using to test my answer?

Step 3 (answers): Trade questions with a partner. Work together to answer the questions. If you are having difficulty with one of the questions, use Step 2 to see if there is another question you could ask that would help you make more progress. Share your question with the class and see if there is another group that has some ideas for you and your partner.

Key Outcomes:

- Communicate where you are stuck.
- Diagnose where you're having trouble.
- Ask specific questions so that the answers you get help you learn.

II. Testing 1,2,3,4

Format: Students working alone, then working in small groups.

Checking your results is an important part of all math problem solving, not just the Guess and Check strategy. There are different ways to check answers and they serve different purposes. This activity is designed to help students think about some of the main approaches to checking one's answer:

1. **Constraints:** Does your result work with all of the constraints in the problem? Review all of the information given and make sure you have tested the result with all of it.
2. **Accuracy:** Have a method for going over your work to test its accuracy: do the inverse operations, reverse your calculations, question your logic and assumptions at each step, etc.
3. **Different Approach:** Solve the problem using a different strategy and see if the results come out the same.
4. **Reasonable:** Can you explain why your answer makes sense or is reasonable using logic or estimates or drawing on experience?

Sample Activity:

Part A: Work individually on Checks 1 and 2 above and prepare to present your work to a small group.

Part B: In a small group, share your work from Part A. Then discuss Check 3. Did anyone use a different strategy? If someone did, compare results. If nobody did, try to come up with a different approach.

Part C: Create a group statement about why the answer you figured out seems reasonable. Is there a way you can show that whatever the answer is, it has to be something like the answer you found?

Key Outcomes:

- Learn multiple methods for checking solutions.
- Develop the habit of looking for constraints.
- Develop the habit of checking arithmetic.
- Develop the habit of not stopping problem solving after finding one possible solution.
- Develop the habit of asking, “is this reasonable?”

III. Hypotheses

Format: Students working in pairs.

Another type of guess and check that mathematicians do is guessing what is the best approach in solving a problem or which information is the most useful in finding a solution. They make a hypothesis and then they test it to see if it works.

Sample Activity:

Step 1 (Hypothesis): Each partner says what they noticed in the problem that they think is going to be most useful or that shows what a good approach will be. Write a sentence like, “I think that this information is really important because _____” or “Because of this information, I think the best approach is to try _____.”

Step 2 (Trial): Each of you then individually tests your hypothesis by using it in a trial run to solve the problem.

Step 3 (Reflect): Share your work with your partner and talk about what you learned from your experiment.

- How did you use the information from your hypothesis?
- Did you get a possible solution, or did you get stuck?
- Did you learn something that you can use for an even better solution?

Note: Students may need to first do Noticings/Wonderings, PoW IQ, or other *Understanding the Problem* strategies before being ready to talk to their partner about what the key information is.

Key Outcomes:

- Pick out and represent key information in the problem mathematically.
- Use key information to generate possible solution paths.
- Monitor whether or not a certain solution path is yielding results.