



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Round 0: Program Description & Introduction

Process standards such as Problem Solving and Communication (NCTM, 2000) represent deep knowledge skills and habits of mind that are the result of an explicit, developmental instructional program, similar to what is done with many mathematics curricula. The Math Forum's Problems of the Week (PoWs) are also designed to support such a program. The Activity Series provides a framework and classroom activities for those who wish to enhance student competence and confidence in Problem Solving and Communication, and to share best practices with other teachers. With each round of the current PoWs, the Math Forum focuses on a particular problem solving strategy along with related communication skill development.

We organize Problem Solving and Communication strategies in a coherent sequence to be studied throughout the year. Each round contains strategy development activities that can be applied to almost any problem including student worksheets with instructions and prompts for students to record their work.

In addition, each round is aligned with several problems from the Problem of the Week Library (available to teacher members with a class level membership or higher). For each of these problems, examples of how the activities might play out in a problem-solving classroom are provided.

The goal is for students eventually to move between multiple strategies flexibly as they attempt to solve problems. Check out the Problem of the Week discussion groups (<http://mathforum.org/kb/forumcategory.jspa?categoryID=6>) for ongoing conversations about Problem Solving and Communication strategies.

Schedule of Rounds

We start by focusing on the Understand the Problem strategy for two rounds, and follow up with three key strategies that can be used both to understand a problem more thoroughly as well as to solve it. We then repeat this cycle, delving a little deeper into ways to use some of the understanding the problem techniques, especially around getting unstuck, finding patterns, and checking work. We follow with Look at Cases and Use Logical Reasoning that help build problem analysis skills and more sophisticated organization and checking of solutions.

After the first eleven rounds we are ready to start bringing different strategies together and connect different mathematical representations in order to support equations, models, and more formal approaches.

Now that students have had some practice executing various strategies, we introduce some activities to hone their thinking about selecting and coordinating strategies. We also explore activities to help students get unstuck and increase their flexibility and creativity in problem solving.

We end the year by returning to where we started with a focus on improving their Wondering skills, learning to pay attention to and stick with the questions that point to interesting problems and solution approaches. Above all, our approach focuses on students experiencing the power of their own thinking, communicating, and collaboration.

The complete list of foci:

- Rounds 1 and 2: Understand the Problem
- Round 3: Guess and Check
- Round 4: Solve a Simpler Problem
- Round 5: Make a Table
- Round 6: Understand the Problem (Revisited)
- Round 7: Guess and Check (Revisited)
- Round 8: Solve a Simpler Problem (Revisited)
- Round 9: Make a Table (Revisited)
- Round 10: Look at Cases
- Round 11: Use Logical Reasoning
- Round 12: Change the Representation
- Round 13: Make a Mathematical Model

Round 14: Work Backwards
Round 15: Plan and Reflect
Round 16: Get Unstuck
Round 17: Play
Round 18: Change the Representation (Revisited)
Round 19: Make a Mathematical Model (Revisited)
Round 20: Wonder

Each year we will release new examples that match the current problems and illustrate the student work we aim for with our activities. We may also revise and tighten the activities slightly over the year; however, last year's documents are still available to all teachers with a class-level membership when they access the problems through the Library. This gives teachers the flexibility to present the activities in a different order, to spend different amounts of time on each activity, etc.

Problem Solving and Communication Activity Implementation

This section provides possible Problem Solving and Communication activity formats for teachers to use every two weeks when new PoWs are released and new Activity Series strategies are featured. Activities may occur over a few problem-solving days during the two week PoW cycle, or could happen over a few minutes a day for the two weeks.

With the exception of the *Understand the Problem*, *Guess and Check II*, *Get Unstuck*, and *Play* rounds, all of the activity series documents follow a basic structure of (1) orienting to the strategy, (2) learning how best to carry out and organize that approach, and (3) reflecting on, comparing, and improving one's strategy. (The excepted rounds instead offer a variety of different stand-alone activities to choose among).

We offer two suggested schedules for working this activity structure into classroom practice.

In just minutes a day:

Day 1: introduce and clarify the featured strategy	5 min
Day 2: notice and wonder and then focus on the first, orienting activity	15 min
Day 3: focus on executing the strategy (activity two)	10 min
Day 4: compare problem solutions (possibly using activity three)	10 min
Day 5: revise solutions	10 min
Day 6: reflect on the featured strategy (possibly using activity three)	5 min
Day 7: complete the problem again, using a different strategy (optional)	10 min
Day 8: create a strategy poster or review/revise an existing strategy poster	10 min
Day 9: apply the strategy to a new problem or create new problems (optional)	10 min

Note: This is a suggested timeline and could be slowed down or sped up as needed. You could vary when you spend the 5-10 minutes on the PoW: it could be a warm-up activity, a short break between activities, at the end of class, or as homework.

As a full class period (or two):

1. Orient to the problem and featured strategy (20 minutes)
 - Scenario 1: Students carry out activities one and two.
 - Scenario 2: Students brainstorm about the featured problem strategy and attempt to use it to solve the problem; as they get stuck or have interest, they try the activities or teacher uses the activities to facilitate the students' problem solving.
 - Scenario 3: Students work on solving the problem however they can, and if they get stuck or after they have tried their own approach, they come back to try one or more of the suggested activities.
2. Discuss the featured strategy: whole class or group discussion and activities (10 minutes)
Students carry out activity three.
3. What else do we want to learn: whole class or small group discussion (10 minutes)
Students develop any questions about what they want to learn or figure out about the featured strategy. These questions along with any insights they have added to their class strategy list can be submitted to the Math Forum for highlight consideration through the PoW teacher discussion group.

Introductory Activities

These activities are designed to orient students to the year's goals and the strands around Problem Solving and Communication in mathematics. We suggest doing at least one activity in each of the following three areas, perhaps spread throughout the first month or so.

Note: These activities are not related to specific PoWs and can be used exclusively without the PoWs (Other documents in the Activity Series will be related to the current PoWs).

I. Problem Solving: Dealing with the Unknown and Unexpected

Activity A: Class Discussion

Brainstorm areas of life in which people prepare to deal with the unexpected and with problems they have not had to solve before (e.g., space exploration, international relations, conflicts, making a movie needing new special effects, global warming, or any area of human endeavor can provide this experience).

Activity B: Describe Your Problem Solving Hero

Individually write/think about a person who is very good at handling new problems. What are the characteristics of such a person? How do they train/prepare in order to get good at problem solving? What do you admire about such a person? What is challenging or hard in their preparation? What makes it worthwhile for them to work so hard to get good at this? Share your description with a classmate.

Activity C: What do Mathematics Problem Solvers Do?

As a class, brainstorm a list of problem-solving strategies that you could use when you do not know how to solve a math problem. Break into small groups, with each group being responsible for starting a class document (e.g., posters on chart paper, blogs, wiki pages, etc.) on one of the specific strategies, including:

- A description of the strategy.
- Illustrations of different ways to use that strategy.
- What it means to get good at that strategy.
- Questions the group has about the strategy.

Note: This activity can be started with students being given PoWs they definitely do not know how to solve. Tell students they are not supposed to know how to solve the problem, or that it is okay to not know how to solve it, you want to use this as a way to stimulate their thinking about all of the things they do to make progress in such situations.

Key Outcomes:

- Understand that problem solving is about having strategies to use when you don't know how to solve a problem and when you want to be creative in your solutions.
- Build personal connections and motivation for getting good at problem solving.
- Establish a way for the class to record, build, and use its knowledge about problem solving over the course of the year. (One possible motivation is that the Math Forum will publish any new methods, approaches, or activities with the name of the submitting class).
- Figure out what your students already know and need to learn about problem solving.

II. Writing to Learn

Wherever we use "writing," one could include "talking" and the broader concept of "accountable talk and/or communication."

Activity A: Class Discussion

Brainstorm ways that writing and talking can help you figure something out or learn something new.

Note: The PoW program will be an opportunity to get good at this. Three themes will be:

- Writing to develop ideas and possible approaches to a problem.
- Writing to figure out what you need to know and good questions to ask to find the answers.
- Writing to be a responsible learner, providing ideas for the group to make use of, and showing ways in which you are using the ideas of the group.

Activity B, Part 1: Create a Plan

Think of something you like to do, but that you would like to get better at. Individually brainstorm some steps you might take to try to improve. The goal is to get your ideas flowing and write them down so you can organize your ideas for yourself, as well as share them with others (e.g., mind map, list, flow chart, etc.)

Activity B, Part 2: Ask an Expert

Think of something you like to do, but that you would like to get better at. Draft an email or letter that you would send to an expert asking them to help you with your "problem." What would you have to do in order

for the expert to understand the help you need? What kinds of questions assist the expert in giving you help that you can really use?

Activity B, Part 3: Share your Plan

Work with a classmate and exchange your “Ask an Expert” email or letter drafts. Take turns role-playing to hear how the “expert” might respond to the questions that were asked. Think about how you initially phrased your questions. Should anything be changed? Your partner could try taking your questions very literally, to help you think about how the expert might respond if they didn’t understand what you meant. Record for the whole class what you learned from your partner’s feedback.

Key Outcomes:

- Understand that writing can help you figure out how to solve problems, especially when you don’t know what to do.
- Identify specific techniques or writing types that can be used to improve problem-solving explanations.
- Students develop a sense of themselves as mathematical writers, with a style and personal connection.

III. Writing to Present and Explain

Activity A: Class Discussion

When you write, you are writing for someone, an audience. Brainstorm as a class different reasons someone might write their solution to a math problem (e.g., to get help, to explain what you did, to show that you learned something, to share something interesting or exciting, to get someone else interested in working with you, etc.). What is needed in the writing to accomplish each purpose?

Activity B: Making a Peanut Butter Sandwich

Write directions of how to do a common task, such as using a paper clip, buttoning a shirt, or making a peanut butter and jelly sandwich for someone who does not know anything about that task (i.e., does not know how or why paperclips are used, or does not know about buttoning shirts nor what buttons are). What happens when your partner follows your directions exactly?

Activity C: What makes a good explanation of a solution?

Look at a sample set of student solution explanations* and identify ways that each one shows a good solution presentation. Also identify what you think is the most important thing each student could do to improve their explanation.

**Student solutions will be available online at the Math Forum once we have prepared student submissions from the first PoW of the year. This activity should be delayed until your class has finished solving and submitting their answers to the year’s first PoW, since it includes full explanations of that problem.*

Key Outcomes:

- Realize that a good explanation makes a difference to an audience and specific features make it better.
- Begin to develop a sense of the progression of writing from giving an answer, to showing what you did, to explaining why you did it and why it works, and to communicating what is interesting and useful about the answer.
- Students begin to develop a sense of themselves as math writers, with a style and personal connection, as well as aspirations to get better at writing mathematically.
- Develop the skill of reading one’s own work through someone else’s eye.

References

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, Virginia: Author.