



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Plan and Reflect

We have presented the following problem-solving strategies:

- *Understand the Problem*
- *Guess and Check*
- *Solve a Simpler Problem*
- *Make a Table*
- *Look at Cases*
- *Use Logical Reasoning*
- *Change the Representation*
- *Make a Mathematical Model*
- *Work Backwards*

As students get comfortable with more strategies, they begin to recognize and use multiple approaches in their problem-solving process. Following Polya (1945), it is common to break the problem-solving process down into four phases: Understanding the Problem, Making a Plan, Carrying Out the Plan, and Checking/Reflecting. Two of the most common issues in problem solving are (1) forgetting part of the problem or the ideas you have discovered that might be useful and (2) getting stuck, trying the same thing over and over without making new progress. Planning and reflecting can help you both solve problems and learn from your experience. What does it mean to get good at making a plan? How do I know when to start carrying out the plan? How do I know if I'm on a dead-end path? How do I effectively check my work?

The activities we have presented so far this year could all be organized into the four stages of problem solving.

1. **Understanding the Problem** (e.g. *Noticing and Wondering* from round 1)
2. **Making a Plan:** (e.g. *Building up or Breaking Down* from round 13)
3. **Carrying out the Plan:** (e.g. *Representing* from round 12)
4. **Checking/Reflecting:** (e.g. *Testing 1, 2, 3, 4* from round 7)

In addition, the structure of most of the activity series documents has been “(understand) plan, do, reflect and revise.” The questions featured in the first activity of each activity series can be helpful in planning and selecting strategies (for example, for *Solve a Simpler Problem*, students asked themselves “what makes this problem hard?”). The middle activities often helped students to organize their work as they carried out the given strategy. Finally, the reflection and revision questions in the final activity offer good ways for groups of students to revise together.

The activities below focus in particular on the planning and reflecting stages, particularly on helping students organize and record for their own and others' use the thinking that they do. These activities may be most useful to students who have learned some of the individual strategies covered earlier in the year.

The activities are written so that you can use them with Problem of the Week of your choosing.

Problem-Solving Goals

Planning helps problem solvers to:

- Use what they notice and wonder effectively.
- Select an efficient strategy.
- Step back from their problem solving and evaluate what they have figured out so far and whether they are on a useful path.

Communication Goals

Problem solvers use writing to help them plan by:

- Writing down what they notice and wonder so that they can make use of their ideas and come back to them if they get stuck.
- Organizing and recording their planning so that they can:
 - Refer back to it easily to get other ideas and remember other parts of the problem solving process
 - Share it with others.

Activities

I. *Choosing a Strategy*

Format: students working alone then moving to teams of 5-6.

Materials: Sticky notes, index cards, or computers with “mind-mapping” software.

The key concepts in this activity are:

- Using Noticings and Wonderings to help choose appropriate strategies
- Making public what Noticings and Wonderings lead to or “go with” a certain strategy
- Comparing strategies before selecting one, discussing which ones could work
- Making public criteria for choosing a good strategy

Sample Activity: Strategy Selection Commission

Part 1, on your own: Notice and Wonder about the problem, and record your noticings and wonderings on sticky notes or index cards, or on a computer so you can re-order them if you want. Re-organizing and sorting your noticings and wonderings can help you choose a strategy you have enough information to use, that fits this problem well. For more ideas of how to sort noticings and wonderings, see the chart in the Student Handouts section.

Choose a strategy or starting approach that you think you have enough information for and that fits the problem well.

- 1) Record the noticings and wonderings that led you to think of this strategy (if you used sticky notes, just stick them on the page!)
- 2) Write any problems you think might happen as you carry out this strategy. You might play out the strategy a little to help you figure out what could go wrong.

If you can think of any other strategies that might work too, record those as well.

Part 2, in your group:

Choose someone to be the organizer for the group. That person will collect what each member wrote about their strategy after they present it.

Each person presents the strategy they thought of to the group, along with what led them to choose it and why they think it would be a good strategy.

After hearing all the presentations, the organizer reminds everyone what the strategies are. Discuss each one and if you think it could be used to solve this problem. If you think it could work, the organizer should put it in a “candidates” pile.

The organizer reminds everyone which strategies are candidates, then group members can vote on the candidates and choose the strategy you want to use to solve the problem. The organizer should put the other candidates in the “Parking Lot” so that if you get stuck you have other strategies to consider.

II. *“Where Are We?”*

Format: students working in their groups of 5-6, or dividing into different teams if there are sub-groups that want to work on a different strategy than their group chose.

The key concepts in this activity are:

- Developing the habit of asking reflective questions during problem solving, and using the reflective questions to inform problem solving (**Trail Marker** role).

- These questions can be asked at regular intervals, when you get stuck, or when you come to decision points or discovery points.
- Key questions are: “what are we doing? why/what do we hope to get out of it? how will we check if it worked? what was the question again?”
- Developing the habit of keeping in mind strategies or Noticings and Wonderings that you considered but aren’t currently using (**Parking Lot Attendant** role).
- Learning questions and tips for getting through moments of being stuck (**Unstuck-er** role).

Sample Activity:

- 1) Decide on group roles:
 - a. **Parking Lot Attendant** – observes the problem solving while looking at the strategy Parking Lot and the Noticings and Wonderings. Is responsible for helping the group use Noticings and Wonderings they may have missed and reminding them of other strategies they considered and what was useful. Participates when asked or when he/she notices something useful.
 - b. **Trail Marker** – observes the group and pauses them regularly to ask them, “What are we doing? Why? What do we hope to get out of it? How will we check if it worked? What was the question again?” Participates every few minutes, plus when he/she notices the group making a decision or trying a new thing. Has a special color pen or marker to record the trail markings.
 - c. **Unstuck-er** – participates with the group, but has the special job of if they get stuck, figuring out who to ask for help and what to ask. Could ask the Parking Lot Attendant what’s in the parking lot, or ask the Trail Marker what they were hoping to do and how they were going to check it, or could decide they need to ask another person, look something up, etc. Can also refer to the strategy-specific.
 - d. **Recorder** – participates with the group, makes sure they write down their problem-solving (though others might right down work they do as well).
- 2) Work on the problem: The remaining group members, along with the Recorder and Unstuck-er, work on the problem together using the strategy or starting approach they voted on earlier. The Trail Marker and Parking Lot Attendant observe. Every few minutes the Trail Marker pauses them and they work together to answer his/her questions. The Trail Marker uses his/her special-colored pen to record their answers on their problem-solving sheet. If the questioning prompts any changes in approach, the Parking Lot Attendant might suggest other strategies or Noticings and Wonderings the group considered.
- 3) Continue this problem until the group has an answer that they are confident in and have checked in some way.

Key Outcomes

- Develop flexibility in carrying out problem solving.
- Develop tactics for getting “unstuck.”
- Increase awareness of multiple strategies and solution paths.

III. Telling the Story

Format: students working in pairs, then each partnership pairing up with another partnership.

The key concepts in this activity are:

- Making space to celebrate and appreciate student thinking and problem-solving
- Learning from what worked and what didn’t work, consolidating and reflecting on the experience of planning
- Exploring the mathematics more deeply by comparing different approaches, and uncovering what was the same about different-seeming approaches and different about similar approaches
- Developing the culture that math isn’t about getting to the answer, it’s about surfacing and exploring more relationships, patterns, and connections

Sample Activity:

Part 1:

With a partner from your group, look back over your plan and what you did as you worked on the problem. Share what you are proud of in your problem solving. Can you figure out what did you did or noticed that led you to those breakthroughs?

Part 2:

Join a pair from a different group. Share your story and what you are proud of. Then, as a group, consider the different strategies that were used. What can you learn about the math of this problem by comparing the different approaches?

Part 3:

In your group of 4, think about what math questions and ideas did you not get a chance yet to explore? What are the interesting noticings and wonderings that you might want to come back to and think about some more?

Key Outcomes:

- Identify ways to improve problem solving.
- Deepen the understanding of the math in the problem.
- Appreciate and hold onto the interesting ideas and questions you came up with.

References

Polya, G. (1945). *How to solve it*. Princeton, New Jersey: Princeton University Press.