



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Play

When students do the *Notice and Wonder* activity, we often have them try to group their noticings into “quantities” and “relationships.” With a little practice, students get adept at finding the quantities and the relationships that are explicitly stated in the problem. However, interesting math problems usually have deeper layers of relationships that only emerge as problem solvers “play” with the relationships and quantities.

In the recent activities focusing on *Plan and Reflect* and *Get Unstuck*, we began to highlight some of the phases of problem solving, and to show how many of the activities in this series can be used to explore relationships as you begin problem solving or if you get stuck along the way.

Continuing in this vein, *Play* focuses on some of the ways problem solvers play with relationships and explore patterns before they delve deeply into a single problem-solving strategy. In order to make clear different aspects of problem solving, we’ve broken the “play” process out somewhat artificially – expert problem solvers move back and forth fluidly between understanding the problem, playing with relationships, and carrying out strategies. However, for purposes of illustration, we think it will be useful to focus on those phases separately.

The activities are written so that you can use them with problems of your choosing. There is no sequence to the activities. Select one or more that seem appropriate or adaptable to your classroom.

Problem-Solving Goals

Good problem solvers:

- Play and explore as they solve problems.
- Look for deeper and hidden relationships.
- Try to uncover more and more interesting math.
- Try multiple approaches or ways of looking at a single problem.

Communication Goals

Problem solvers use communication as they play to help them:

- Keep track of interesting things they noticed and wondered.
- Represent the problem in new ways.
- Paraphrase the problem.
- Share their own perspectives and ideas and learn from others.

Activities

I. Calculating (and Noticing) as you Go

Sample Activity: Calculate as you Go

Format: Students working in pairs.

When you see quantities in the problem, you may not see how to solve the problem, but you might think of some calculations you could do. Try doing some of these calculations that come into your head, even if you don’t know that they will help you solve the problem.

Be sure to tell your partner:

What you did (what quantities and operations you used).

What the units of the results are (what you are counting or measuring).

As you calculate, notice if you get any interesting results, or if any of the calculations seem particularly helpful.

II. *Playing with Strategies*

Sample Activity: Speed Dating

Format: students working in groups of three to five.

Materials: strips of paper, pens or pencils, loose leaf paper.

In order to get the juices flowing and begin to investigate and unearth more relationships, it can be helpful to try a lot of different ideas quickly. See what you notice, but don't get too bogged down in one idea.

- 1) Each person writes a strategy or short description of something to try on a strip of paper. The activity will be more fun if each person chooses a different strategy. Some good examples: *Guess and Check*, *Change the Representation*, *Make a Table*, and *Solve a Simpler Problem*.
- 2) When everyone is ready, each member of your group should pass his/her strip of paper to the left. You have three minutes to do what you can with the strategy or idea that you received. Write your work and what you notice and wonder on your own sheet of loose-leaf paper (this way, at the end, you will have ideas from a few different strategies that you can look back at as you work on the problem).
- 3) After three minutes, stop wherever you are and draw a line or a box around your work, and write the name of the strategy used.
- 4) Pass the strategy strip you were working on to your left, and receive a new one on your right. You have three minutes to work on the new strategy. Keep working on your own paper with a new section for each strategy that you do.
- 5) Pass papers every three minutes until you receive the strategy strip that you originally wrote. Finish by working on that strategy for three minutes.
- 6) As a group, add any new relationships, patterns, quantities, interesting ideas, or things you are wondering about to your list of noticings and wonderings.

Key Outcomes

- Get a better understanding of the problem by playing with a variety of ideas before solving the problem.
- Identify and pull together the most promising solution path from multiple representations or multiple strategies.
- Discover and note deeper, hidden relationships that emerge as you play with various possibilities.

III. *Playing with Clues*

Sample Activity: What If...

Format: students working in groups of three to five.

"Clues" is shorthand for the longer phrase "quantities and relationships you noticed or wondered about."

One way to understand how a particular clue is a useful part of the problem is by changing it and noticing how the problem changes. I might change the value of a clue, or even pretend I don't know it. My goal is to focus on what changes in the problem, and how I can use that information to understand the clue better.

- 1) With your group, go through each clue or set of clues. Write down ways you could change the value of quantities in that clue or the constraint that it imposes.
- 2) For each clue, play out the problem a little bit with the new value(s). How would the problem change? Would it be easier or harder? What would be easier or harder about it? Would the results be different?
- 3) Now go through the clues again, this time ignoring one clue or set of clues at a time, pretending that information was never given.
- 4) How does the problem change when those clues are ignored? Does it make a simpler version you can use to learn more? Does it change the number of possible answers? What else changes?
- 5) After you've gone through all the clues, look back at the original problem. What new understanding have you gained? Do you see more uses for any of the clues? Do any of the clues seem more necessary (or unnecessary)?

Key Outcomes

- Explore how the problem was constructed.
- Generate additional information and perspective by changing or ignoring clues.
- Gain better understanding of the problem by thinking about simpler (and harder) versions.

IV. *Playing with Representations*

Sample Activity: How Else Can We Say It?

Format: students working in groups of three to five (depending on the number of clues in the problem).

Sometimes the clues are said one way, but if you said them (or wrote them or drew them) just a little differently, you would see different relationships. In this activity, try to express the clues as many different ways as you can.

- 1) Each person picks one clue from the problem and rewrites it or draws a picture of it or somehow changes *how* it's said without changing *what* is said.
- 2) After a few minutes, each person passes the clue to their left. Try to add another way to say (or draw or represent) the clue, adding to those already written down.
- 3) Keep passing clues to the left as long as you can come up with new ways to express them.
- 4) Once you've run out of ideas working individually, hold a group discussion:
 - a. Check if any of the different expressions changed *what* the clues really meant. In this activity you don't want to change what the clues mean. You just want to get a new perspective on what they mean.
 - b. Did any new information or perspective emerge that helps you see an approach to solving the problem?

Key Outcomes

- Understand the problem better using multiple representations of key information.
- See the problem from a fresh perspective.