The Math Forum: Problems of the Week

Problem Solving and Communication Activity Series

Understanding the Problem II

In this revisiting of the Understanding the Problem strategy, we have updated the activities to help students deepen their focus and improve their problem-solving skills. In this document, you will notice:

- We added a "parking lot" to the Noticings/Wonderings activity. (Enhancement of Round 1 Activity!)
- We focus on "implications" in the Extracting the Information and Question activity. (Enhancement of Round 1 Activity!)
- We focus on "key words" in the Paraphrasing activity. (New Activity! See Round 1 or 2 also)
- We focus on mathematically-dense drawings in the Act It Out activity (New Activity! See Round 1 or 2 also)

The activities are written so that you can use them with problems of your choosing.

Problem Solving Goals

Different techniques for understanding the problem can lead us to ideas for solving a problem we have never used before. Good problem-solvers use this problem-solving strategy and may come back to it often as they're working on the problem, to refine their strategy, see if they can find better solutions, or find other, even more interesting questions. Specifically, good problem-solvers:

- Use various methods to make sense of a problem from different perspectives.
- Pull out the relevant information.
- Connect to prior knowledge and experience.
- Represent problems in multiple ways.
- Focus their representations on mathematically-relevant information.

Communication Goals

Writing is an integral part of understanding the problem and builds momentum in thought. It helps the problem solver organize their information and articulate the questions they will address on their journey towards the solution. Specifically, when trying to understand the problem, good problem-solvers might:

- Create an organized list of what they noticed and what they wondered about the problem.
- Record implications or background knowledge that they apply to a problem.
- Paraphrase the problem in their own words.
- Make clear drawings and well-labeled diagrams.

Activities

I. Noticings/Wonderings

Format: big-group brainstorming or go-round activity.

Step 1: Make a list of all of the mathematical information and relationships you notice about the problem, and everything you wonder about the problem. Your noticings may include:

- The quantities that can be counted or measured.
- Relationships between quantities.
- Information that is not given in the problem but is made relevant by it.
- Key words from the problem.

Your wonderings might take the form of:

- I wonder what will happen if …
- I wonder what this word means …

- I wonder if this pattern will continue …
- I wonder whether …

Sample Activity: Noticings/Wonderings and the Parking Lot

Step 1: As a whole class, or in a few groups (5 or more students), have students go around quickly and offer one mathematical relationship or bit of mathematical information that they notice, or a mathematical question they are wondering about. Record each noticing and wondering. Continue around the group until no one has anything more to offer.

Prepare students to write down their reactions as the group is reporting their noticings/wonderings. They may be having reactions like:

- Wow, I never noticed that!
- I wonder if ____ is important to the problem.
- I noticed ____, which "answers" the wondering ____.
- I wonder if you really need to figure out ____ to solve the problem.

Rather than share those ideas during the brainstorming, jot them down or remember them for Step 2.

Step 2: Discuss as a class or in their group, the following questions:

- How did other people think about the problem differently from you?
- What assumptions did they make?
- What's different from how you thought about it and what you noticed?

Step 3: Make a "Parking Lot." As you notice and wonder about the problem, which issues need to be resolved? Are there ones you can put aside? Are there issues you can "agree to disagree" about? Write those issues on a separate list, called the "Parking Lot." Once you have solved the problem, refer back to the parking lot to see if you resolved the issues, or if you need to explore them further.

Key Outcomes

- Student ownership and understanding of the question to be solved.
- Momentum toward a solution path stimulated by all of the mathematical quantities and relationships noticed.
- Slow down the thinking process and surface all of the information and questions that are too easily passed over or dismissed.
- Articulation of specific sub-problems or questions students need to answer or learn more about in order to solve the problem.
- Identify students' own and others' assumptions.
- Identify assumptions that can appropriately simplify the problem.
- Identify other questions and features of the problem that may be even more interesting and challenging.
- Distinguish between more and less relevant information.

II. Extracting the Information and Question

Format: whole group brainstorming, individual brainstorming, or think-pair-share.

List the key mathematical information that may be useful in solving the problem as succinctly as possible and state what will count as an answer.

- Identify and list important information given in the problem:
    - What quantities are given?
    - What terms are important?
    - What constraints are given?
- Write down implications of the given information:
    - What do you already know about the key terms or ideas?
    - What calculations could you do?
    - What relationships can you describe?
- Predict as much as you can about the final answer:
    - What will the units of the solution be (what will be counted or measured)?
    - What justification is needed/what am I trying to prove?
    - Can I figure out upper and lower bounds?
    - Could the answer be negative? Could it be a non-integer?

Sample Activity: PoW IQ

"PoW IQ" stands for extracting the **I**nformation in the problem, and understanding the **Q**uestion.

Have each student make a table with very compact mathematical information from the problem in the left column, calculations or mathematical relationships they see in the middle column, and questions they should explore for the final answer in the right column. **[See student handout below for table template].**

Key Outcomes

- Student ownership and understanding of the constraints a full solution requires.
- Articulate mathematical information in a simple, compact format that makes patterns and relationships visible and moves students toward possible solution paths.
- Make visible the background knowledge that students bring to a problem.

III. Paraphrasing

Format: think-pair-share or students working individually.

The goal of paraphrasing a problem is to have students analyze the language of a problem and make clear the mathematics behind the problem situation. Some prompts you might use with students are:

- Identify the key words/phrases in the problem. How would you define them?
- How would you rewrite the problem?
- Same math idea/Different math story: How could you put the problem in a different scenario, while preserving the math behind the problem?

Sample Activity: Key Words

Highlight words or phrases you think are going to be really important to understanding and solving the problem. Write each word on a sheet of paper, with several lines between each. Below each word, sketch or write what you think it means. Then write what you need to find out about that word, or as many things as you can think of that the word tells you about the problem.

Key Outcomes

- Learn to make sure the problem makes sense to you and that you know what has to be figured out.
- Put your thoughts in writing so you can compare your thinking to the original problem statement and see what you may be missing or changing without realizing it.

IV. Acting it Out

Format: small groups.

This approach often requires the most teacher/expert support to ensure the tools or manipulatives that support the investigation are available, and that students are making sure their modeling of the problem fits the necessary constraints. Some examples include:

- Physically acting out the problem by using actual materials from the problem situation or using virtual manipulatives (You might act out a simpler version of the problem, for example, using smaller numbers).
- Drawing a rough sketch (This is different from drawing a picture as a strategy to solve the problem, since you know you might be drawing it imperfectly, but you are just trying to get a sense of relationships).
- Doing the problem "wrong": similar to a quick version of guess and check, doing the problem wrong can refer to working through the problem by guessing a number for an unknown quantity, or trying to find an answer that works without being sure you have found every possible answer. In either case, the focus is on understanding the problem scenario and key relationships, rather than trying to get a full solution to the problem.

In the sample activity below, the focus is on the concept of a rough sketch. What visual information is important to include? How can that information be simplified or abstracted? What are effective ways to use labels? How accurate does the sketch have to be?

Sample Activity: Rough Sketches

Step 1: Imagine a movie or photograph in your head of the problem. Imagine the story of the problem or the perfect diagram for it. Think of what in the movie or photograph you could count or measure. Try to imagine removing from the movie or photograph all of the "extra" non-mathematical stuff. What's left?

Step 2: Draw a rough sketch of what was left. Represent what you can count or measure, and leave out as much of the "extra" information as you can. Label your sketch with numbers, letters, or words if you can.

Step 3: Share your drawing with your partner or group. Compare your drawing with the other drawings. What are the similarities? What are the differences? What countable things do you see? What measurable things do you see? What would you label? What might you do differently?

Step 4: As a group, try to make the simplest, clearest sketch possible. Use it as you try to solve the problem. How is it helpful?

Key Outcomes:

- Use visual and physical intelligence to develop a sense of what is going on in the problem.
- Figure out an answer or a good estimate and use this to start thinking of explanations about why it works that way.
- Develop techniques for representing information abstractly.
- Become more comfortable with drawings that are not to scale or not representative.